**Indian Statistical Institute, Bangalore Centre**

B.Math (Hons.) II Year, Second Semester

Mid-Sem Examination

Algebra IV

Time: 9.00AM-1.00PM March 04, 2013 Instructor: Bhaskar Bagchi

Maximum Marks : 100.

1. In each of the following cases, find the minimal polynomial $f \in Q[X]$ of $\alpha$, the splitting field $E$ of $f$, and the Galois group of $E$ over $Q$.

   (a) $\alpha = 1 + \sqrt{2}$ (b) $\alpha = \sqrt{2} + \sqrt{3}$, (c) $\alpha = \sqrt{3 - 2\sqrt{2}}$ ,
   (d) $\alpha = (1 + \sqrt{-3})^{1/3}$ [20]

2. Let $E \supseteq F$ be finite fields. Then compute the Galois group $Gal(E, F)$ in terms of the orders of $E$ and $F$. Hence, show that this is a Galois extension. [20]

3. Let $E \supseteq B \supseteq F$ be finite extensions of fields. (a) If $E \supseteq F$ is Galois then show that $E \supseteq B$ is Galois. (b) If $E \supseteq B$ and $B \supseteq F$ are both Galois then show that $E \supseteq F$ is Galois. [20]

   (Hint : May use any of the criteria proved in class for an extension to be Galois).

4. Let $f$ be an irreducible polynomial over a field $F$. If $\text{char}(F) = 0$ then show that $f$ is separable. Given an example, with $\text{char}(F) = 2$, where $f$ is not separable. [20]

5. Show that, for any field $K$, $\text{Aut}(K)$ is a linearly independent subset of the $K$- vector space $K^K$. [20]